"Plot 1 is All Spread Out and Plot 2 is All Squished Together": Exemplifying Statistical Variation with Young Students

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The idea of variation is a foundation of statistical reasoning, and many curriculum documents, including the *Australian Curriculum*, include variation in the learning required for the primary years. In this paper, we consider the design of activities that can exemplify the idea of variation for young students and investigate how students can use graphs to support discussions about variation. The use of appropriate contexts and the provision of physical experiences of the phenomena seemed to help students make sense of graphical representations and allowed them to discuss how variation was exemplified in the graphs.

Many phenomena involve attributes that are different under different circumstances. For example, men are, in general, taller than women (here the height attribute varies across two groups); within a group of women the heights of individuals will vary (here the height attribute varies within a group); and for a particular woman, her height may change over time (and here the height attribute varies temporally). The field of statistics, which has issues of variation at its heart, provides tools for dealing with such phenomena. The *Australian Curriculum: Mathematics* (*ACM*) acknowledges the role of variation within the Statistics and Probability strand and, beginning at Year 3 level, includes reasoning with data that involves "interpreting variations in the results of data collections and data displays" and continues through to allowing for the variation that might be present when determining lines of best fit for a scatter plot in Year 10 (Australian Curriculum, Assessment and Reporting Authority, 2018).

With variation a core concept for statistical understanding, it is important to investigate what activities allow students to learn about variation as a data phenomenon, and how certain representations can exemplify variation in meaningful ways. For young students it is vital to identify activities that build a foundation for understanding variation. The study reported in this paper had, as its focus, an examination of the kinds of experiences and activities that will allow students to make sense of variation, and whether or not students can make sense of and use representations that might help exemplify variation.

Background

Statistical Variation

As Moore (1990) pointed out, without variation there would be no need for statistics. Traditionally in the school mathematics curriculum, however, expectation, based on averages, has received more attention. Shaughnessy (1997) suggested this may have been related to the formula for the arithmetic mean being much easier to calculate than the formula for the standard deviation. Although much early research in statistics education

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followed this line (e.g., Strauss & Bichler, 1988), when classroom research began, it became evident that children's basic appreciation of variation emerges before their appreciation of expectation (Watson, 2005). That the *ACM* now recognises this, and includes reference to variation in year 3, puts pressure on primary teachers to provide meaningful experiences that exemplify the concept. Nevertheless, although the curriculum talks about variation for young students, it does not make clear the scope of what might be learned about with respect to variation.

There are three types of variation situations that are of interest in this study. The first, variation within a group, concerns situations where there is a single group and an attribute/variable that varies among cases in the group. An example of this might be measuring how far students' paper planes can fly: some will fly only a short distance while others fly further. The second situation is variation across groups, where there are different groups and one or more attributes/variables that are being considered and there are differences from one group to another. Comparing how much pocket money year 6 students earn with how much pocket money year 3 students earn is an example of such a situation. It should be noted that there will almost certainly be within-group variation as an additional phenomenon in these cases. The final situation is variation with time, in which an attribute may vary as time passes. As an example, consider how the height of a bean plant changes with time. This phenomenon may also arise in conjunction with within-group and across-group variation. These types of variation are familiar to those who study formal statistics at the tertiary level (e.g., Moore & McCabe, 1993); the question is whether they can be exemplified and comprehended in the primary years.

Exemplifying

To help students learn general principles they are often given examples that are specific instantiations of the principle, with the expectation that the general ideas will become evident as they work with the specific example/s. For instance, to learn about outliers and how they affect the mean, a teacher might have students examine a certain set of house prices as a specific illustration. As discussed in Chick (2007), one of the critical roles for a teacher is to choose and use examples that allow students to learn the intended principle. This involves being able to identify or design for key affordances (Gibson, 1977) within the chosen example and then use it in the classroom, so that the example succeeds in exemplifying the principle. Achieving this can be difficult; Chick and Pierce (2012) showed that preservice teachers struggled to design lessons that effectively used a specific data set that had many affordances for teaching general statistics principles.

In addition to the issue of using situations to exemplify general concepts, there is another kind of exemplifying that is relevant for this paper. This concerns how students exemplify the evidence underpinning assertions that they make. Among studies with upper primary students, Watson and Moritz (1999) found that students could use supplied graphical representations to exemplify or support their assertions about differences between two classes' maths scores. Chick and Watson (2001) examined how students could use their created graphs to exemplify claims they made about the data, finding that some students could exemplify the situation with sophisticated representations that they interpreted effectively. There has been less work in this area with younger students.

Research Aims

This paper focuses on some of the issues surrounding the design of scenarios that allow exemplification of the concept of variation, and how to make the concept visible to young students. In addition, the report explores whether students can recognise that graphs might exemplify the variation in phenomena they have experienced, and how they talk about what they see in the scenarios and the graphs.

Scenarios for Exemplifying Variation

The two scenarios to exemplify variation used for this report were devised in the first year of a 4-year research project using data modelling to enhance STEM in the primary curriculum. Previous research (e.g., Watson, 2005) suggested it was essential to make variation explicit for students, a phenomenon they experience daily in many diverse ways.

Licorice scenario

The licorice scenario was adapted from earlier work with teachers and young children (Watson, Skalicky, Fitzallen, & Wright, 2009). Students used Play-Doh™ to make licorice sticks in two ways: by hand and with a Play-Doh™ Extruder. The requirement was that the sticks be 8 cm long and 1 cm in diameter. Before students began making the sticks with this "factory" and by hand, the word "variation" was introduced, and examples from the students' experiences discussed. To compare across the two ways of making the sticks, all sticks (three of each type per child) were weighed and their masses recorded on sticky-notes. Within each method, the variation in the masses was discussed, with students giving reasons for its occurrence (e.g., care with the ruler, reading the scale carefully). The researchers then needed to represent the information visually to reinforce the variation within each method's data and the difference in the variation between the two methods, for students who had, until then, only experienced bar graphs as a form of data representation.

Heat scenario

The second activity was based on the concept of heat in the Year 3 Science curriculum, using the measurement of temperature and elapsed time, which were new experiences for most students (Fitzallen, Watson, & Wright, 2017). The concepts covered were more complex than for the Licorice activity. Insulated and non-insulated plastic cups were filled with hot water and placed in a trough; measurements of the temperature in the two cups were taken every 5 minutes for 30 minutes; 10 minutes from the start, ice water was added to the trough and its temperature was also measured for the remaining 20 minutes. Students recorded the temperatures in a table and described the change in workbooks. The question for the researchers was how to represent the variation present for the students who had no experiences with Cartesian graphs; the resolution is shown in the Results section.

Method

Participants

The activities were carried out in two year 3 classes (students about 9 years old) in a parochial school in Tasmania, with data collected from 48 students for Licorice and 49 students for Heat. The teachers taught the lessons for the activities with notes provided by the researchers. Four members of the research team (including the second and third author) were present at each activity to assist with the materials and supervision of the group work.

Data Sources

The comments about students’ interactions with the actual physical situations—making licorice and observing changes in temperature—are based on the field observations of the research team (including the second and third authors). For the licorice scenario, the students had a workbook with guiding questions for their observations during the activity. After making the sticky-note plots shown in Figure 1, one workbook question asked them to list the differences between the two plots. Their responses provided the data for some of the results. Data for the heat scenario came from transcripts of class discussions recorded on video. These class discussions had input from both teachers and researchers.

Results

For each scenario, we will first report on the students’ reactions to the tasks. We then consider how they described and interpreted the variation from graphical representations.

Licorice scenario

Some students were already familiar with the Play-doh™ Extruder and there was great interest in the activity. Students were very careful in making the hand-made licorice sticks, trying to achieve even thickness and equal length. Students used electronic scales to record the masses of the licorice sticks, which provided the data for their discussion comparing the factory-made and hand-made situations.

As seen in Figure 1, stacked “dot” plots were used, with students selecting a sticky-note with one of their data values and placing it on the plot. The teacher began by asking for the heaviest and the lightest masses and used them to determine the scale on the axis. As this was a new experience for the year 3 students, guidance was needed at the start, both in working out the distances between the labels and in creating the stacks.

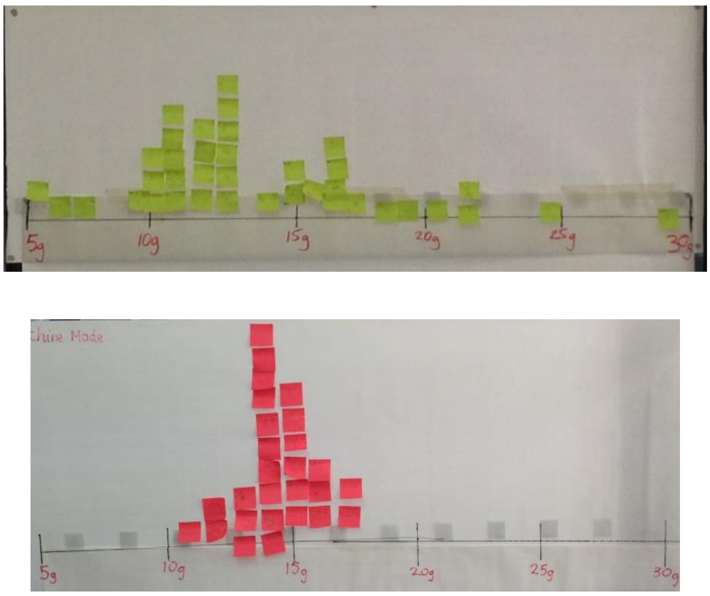

Figure 1. The stacked sticky-note plots, showing the mass data from the hand-made licorice sticks (top) and the factory-made licorice sticks (below).

The students were asked to list any differences between the two plots in Figure 1. One student did not clearly refer to the graphs to discuss the variation, but certainly identified the variation that he/she saw between the groups [spelling errors and punctuation have been corrected in all quotations]:

> The ones that we made with our hands were very different but the ones the machine made were a little bit different.

Other students talked about the spread of the values in the graphs, and the way these varied, using age-typical language that captured the contrasting ideas of "spread out" and "compressed", as illustrated by a sample of such responses below.

> The hand-made class plot is very spread out with the masses. As for the factory-made class plot, all the masses were mainly the same but are a bit different.

> The hand-made one was spread out and the factory-made one was stacked on top of each other.

> Plot 1 is all spread out and Plot 2 is all squished together.

> The first one [hand-made] is like all around the place and the second one is straight in the middle.

> They all are spread out on the handmade one, and all are bunched in on the machine-made one.

Another set of comments additionally referred to specific values from the data, often giving an indication about modal values or the range.

> The one with the machine most were the same, the others were different. When you make it handmade it is bigger. Handmade one has 28 as the highest.

> One was in between 10g and 16g and the other was in between 6g and 28g. The machine is more accurate.

> There are more different weights in the handmade one. Most people have 14g on the machine-made one. Nobody on the machine-made one had 5g and one person on handmade did.

Some students described the graphs in terms of buildings and described the variation in terms of the shapes of the sets of buildings.

> The handmade is like a city with homes spread out and factory-made is the same except the factory-made is with tall buildings, not many but tall buildings.

> (1) 14 is more common in machine-made. (2) In machine-made they are more closer together than handmade. (3) Handmade is spread out and machine is more close together. (4) Handmade is like a city but machine-made is like a tower.

Finally, some students seemed to have picked up on the language used by the teachers, describing the differences between the graphs using the word "variation" itself.

> Factory-made had a larger typical number. Hand-made had more variation in their mass.

> There aren't many variations with the factory-made licorice and there is a large variation with the hand-made licorice.

> The plot with the machine-made things go straight up. Machine-made things don't have much variation.

In the quotations above it is evident that students not only noticed that the two groups were different, but that it was the contrasts in distributions—or the differences in the ways each group varied within the group—that characterised the difference. Moreover, students were able to refer to features of the graphs to exemplify the variation between the groups.

Heat scenario

The heat scenario was much more complex than the licorice one, with variation evident over time, across the three conditions, and across the data from different student groups. Because students had used thermometers to collect the data, it was decided to provide a plot made up of thermometers (a y-axis) for each time (x-axis) that measurements were made (e.g., see Figure 2). Students finally transferred the data to the graph in Figure 3.

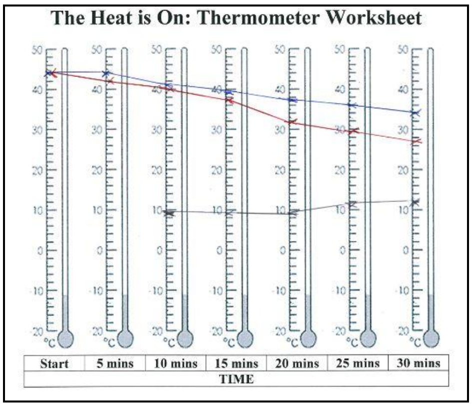

Figure 2. Graph created to show the temperature variation over time for one student group: top line is insulated cup, middle is plain cup, and bottom line is the ice water in the trough (note the use of repeated representations of a thermometer to record the temperature values over time).

The class plot of the data from each group (Figure 3) served two purposes. First, it was a way to include all students in the summary, as each group member put up the dots for one set of measurements from the group. Second, it showed the "between" group variation across groups. Although there was discussion about differences among the groups—for example, the number of ice cubes making some water mixtures colder or judging the time exactly for the measurements—the main aspects of variation that students discussed were changes over time and the difference between the insulated and non-insulated cups.

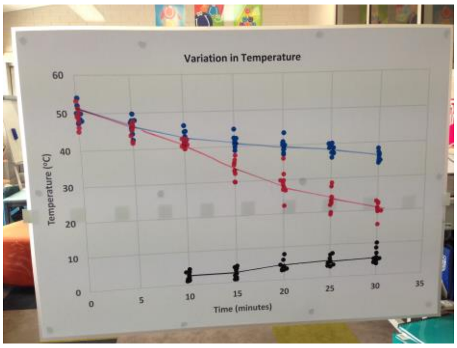

Figure 3. Graph showing the results of the heat experiment for all the groups, showing variation among conditions (top line: insulated cup, middle: plain cup; bottom: ice water), over time, and across groups (dots).

Students were able to talk about the variation that they noticed over time, together with the variation that they noticed across the conditions (plain cup, insulated cup, ice water), and could do so in reference to the graph. This is illustrated by the following quotations from the class discussion in response to various teacher and researcher questions.

> The blue [Fig 3, top line, insulated cup] has gone … staying … and a bit lower. And the red [middle line, plain cup] has gone straight down and the same bit and the cold ice has gone same and up.
>
> That the black – the black line with the – the ice water was the most consistent […] And the non-insulation was the most [long pause] – had the most variation.
>
> That there's a lot more variation [for the red line] than the blue line.
>
> It [the red line] goes down a little bit more than the blue.

When questioned about the variation across students' groups (resulting in different temperatures for the same time and conditions), students offered plausible explanations.

> Because from 10 minutes to 30 minutes the ice could have – there could have been more ice in one to make it colder and less ice when they get melted.
>
> Because of variation if he [the researcher adding hot water to the cups] came around, tipping the water in the thing [trough] he could have […] put a little bit more in one and less in the other.
>
> Some of the water could have been out of the [heating jug] longer than other parts – other could have been a bit warmer than the one that you poured out first – could have cooled down a bit quicker.
>
> Also, because are sitting near the windows and cool air comes …

The students' comments indicate that they had noticed the obvious changes with time, and the differences in cooling for the plain cup and the insulated cup (and the trough). In addition, they were able to recognise that there was variation across the data from different groups' experiments and offer sensible reasons for this.

Discussion and Conclusion

For those experienced in reading graphs it is obvious that the situations exemplified variation: Figure 1 clearly shows variation within the groups and a difference in variation between the two groups (hand-made and factory-made); Figure 2 shows variation among three conditions (plain cup, insulated cup, and iced water) as well as variation with time; and Figure 3 shows, in addition, the variation among data obtained by different groups of students (the different dots around each time point for a given condition). The careful design of the activities and emphasis on variation as the key focus of the activities also allowed this variation to be evident to year 3 students. Despite their young age, they could notice variation over time, variation among situations, and, very specifically in the licorice task, variation (i.e., differences) in the within-group variations present in each of the two groups/situations. Students were also able to notice and explain the variation across different groups of students in the heat scenario. Their awareness of these aspects of variation was governed by the variation-inherent activities and the in-class discussions that they had. They were able to use their own and newly-learned language to describe the differences that they saw.

Interestingly, later statistical work often focuses on differences in means, and the importance of potential differences in the variation among groups gets lost in the formulas. In the licorice activity the expected "typical" value was the same for each method; here the difference between the factory-made and hand-made cases really is made evident by observing the differences in variation between the two groups. There are many statistical situations where it is the variational difference that matters; this research demonstrates that even young students are capable of noticing and making sense of such differences.

The success of the activities relied not only on the exemplifying power of the activities themselves but on the way in which discussions were scaffolded (see Chick, 2007, for more about teacher knowledge for successful implementation of examples). Importantly and additionally the graphs could be and were used to exemplify the phenomenon. This required a clear focus for the teachers on variation as a learning outcome. There can be risks associated with the implementation of such activities where the focus can diminish to the "fun" of the activities and fail to progress beyond this to the variation and graphical ideas. Chick and Pierce (2012) suggest that, in teaching, there can be a tendency to do the active hands-on part of the work, but not have a deep consideration of the concepts. In this case, however, the tasks seem to have been implemented in a way that allowed exemplification of a number of different types of variation, and also empowered students to exemplify or give evidence of the variation that they observed. Although the tasks had complex, multi-faceted aspects most of the students were able to talk about variation in meaningful ways and saw variation as a concept that allowed them to describe change and differences in data-rich situations.

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